JEE-Main-27-01-2024 (Memory Based)
[EVENING SHIFT]

Mathematics

Question: If 20th term from the end of progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ... 129\frac{1}{4}$ is _____.

Options:
(a) -120
(b) -115
(c) -125
(d) -110
Answer: (b)
Solution:

$$T_{20} = a + (n-1)d$$
$$= -129\frac{1}{4} + 19 \times \frac{3}{4}$$
$$= -115$$

Question: $P = (1-x)^{2008}(1+x+x^2)^{2007}$; find the coefficient of x^{2012}?

Answer: 0.00
Solution:

$$(1-x)^{2008}(1+x+x^2)^{2007}$$
$$= (1-x)(1-x^3)^{2007}$$
$$= (1-x)\sum_{r=0}^{2007} {}^{2007}C_r (-x^3)^{2007-r}$$

Coefficient of $x^{2012} = 0$

Question: The integral $\int \frac{(x^8 - x^2)}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)} dx$ is equal to:

Options:
(a) $\frac{1}{3}\ln\left|\left(\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right)\right| + C$

(b) $\ln\left|\left(\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right)\right| + C$

(c) $\frac{1}{6}\ln\left|\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)\right|+C$

(d) $\frac{1}{9}\ln\left|\left(\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right)\right|+C$

Answer: (a)

Solution:

$$\int\frac{\left(x^8-x^2\right)}{\left(x^{12}+3x^6+1\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}dx$$

$$=\int\frac{x^6\left(x^2-\frac{1}{x^4}\right)}{x^6\left(x^6+3+\frac{1}{x^6}\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}dx$$

$$=\int\frac{\left(x^2-\frac{1}{x^4}\right)dx}{\left(\left(x^3+\frac{1}{x^3}\right)^2+1\right)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}$$

Put $\tan^{-1}\left(x^3+\frac{1}{x^3}\right)=t$

$$\Rightarrow\frac{1}{1+\left(x^3+\frac{1}{x^3}\right)^2}\times3\left(x^2-\frac{1}{x^4}\right)dx=dt$$

$$\Rightarrow\frac{1}{3}\int\frac{dt}{t}=\frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|+C$$

Question: $S_n=2023\alpha^n+2024\beta^n,\ x^2-x-1=0$

Options:

(a) $2S_{12}=S_{11}+S_{10}$

(b) $S_{12}=S_{10}+S_{11}$

(c) $S_{12}=S_{10}+S_{11}$

(d) $2S_{12}=S_{10}+S_{11}$

Answer: (a)

Solution:

$$ax^2+bx+c=0$$

$$aS_n+bS_{n-1}+cS_{n-2}=0$$

$$S_{12}-S_{11}-1\times S_{10}=0$$

Question: Consider an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$; eccentricity e_1 & Hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, with eccentricity e_2, $e_1e_2=1$. Ellipse passes through the foci of hyperbola find the length of the ellipse along $y=2$.

Answer: $\frac{10\sqrt{5}}{3}$

Solution:

Hyperbola

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

$$e_1=\sqrt{1+\frac{9}{16}}=\frac{5}{4}$$

For ellipse

$$2a=2\times 4e_1$$

$$a=4e_1$$

$$a=5$$

Also

$$e_1e_2=1$$

$$\frac{5}{4}e_2=1$$

$$e_2=\frac{4}{5}$$

By ellipse formula

$$b^2=a^2\left(1-e_2{}^2\right)$$

$$b=3$$

Ellipse : $\frac{x^2}{25}+\frac{y^2}{9}=1$

Put y = 2 in ellipse equtión

$$x_1{}^2=\frac{25\times 5}{9}$$

$$x_1=\frac{5\sqrt{5}}{3}$$

Length of the chord

$$\Rightarrow l=2x_1=\frac{10\sqrt{5}}{3}$$

Question: If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in $\left[0, \frac{n\pi}{2}\right]$ least value of $n \in N$, then $\sum_{k=1}^{n} \frac{k}{2^n}$ is equal to _____.

Options:

(a) $\frac{9}{2^9}$

(b) $\frac{91}{2^{13}}$

(c) $\frac{7}{2^7}$

(d) $\frac{11}{2^{12}}$

Answer: (b)

Solution:

$2\tan^2\theta - 5\sec\theta = 1$

$2\left(\sec^2\theta - 1\right) - 5\sec\theta = 1$

$2\sec^2\theta - 5\sec\theta - 3 = 0$

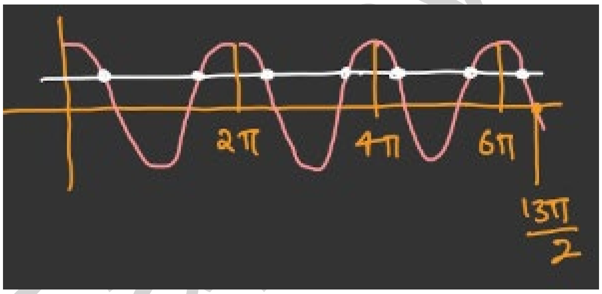
$\Rightarrow \left(2\sec\theta + 1\right)\left(\sec\theta - 3\right) = 0$

$\Rightarrow \sec\theta = \frac{-1}{2}, \sec\theta = 3$

$\cos\theta = -2, \cos\theta = \frac{1}{3}$

7 solutions in $\left[0, \frac{n\pi}{2}\right]$

$n = 13$



$\sum_{k=1}^{13} \frac{k}{2^{13}} = \frac{1}{2^{13}}\left(1 + 2 + ... + 13\right)$

$= \frac{1}{2^{13}} \times \left(\frac{13 \times 14}{2}\right) = \frac{91}{2^{13}}$

Question: $\lim_{x \to 0} \frac{3 - a\sin x - b\cos x - \log\left(1 + x\right)}{3\tan^2 x}$ is non zero finite. Find $2b - a = ?$

Answer: 7.00

Solution:

$$\lim_{x\to 0}\left(\frac{3-a\sin x-b\cos x-\ln(1+x)}{3\tan^2 x}\right)$$

$$\lim_{x\to 0}\frac{\left(3-a\left(x-\frac{x^3}{6}\right)-b\left(1-\frac{x^2}{2}\right)-\left(x-\frac{x^2}{2}\right)\right)}{3x^2}$$

Coeff. of $x^0: 3-b=0 \Rightarrow b=3$

Coeff. of $x: -a-1=0 \Rightarrow a=-1$

$\Rightarrow 2b-a=7$

Question: $\int_0^{\pi}\frac{dx}{1-2a\cos x+a^2}$ is equal to:

Options:

(a) $\frac{(1+a^2)\pi}{(1-a^2)^2}$

(b) $\frac{\pi}{(1-a^2)}$

(c) $\frac{(1-a^2)\pi}{(1+a^2)}$

(d) $\frac{(1-a^2)\pi}{(1+a^2)^2}$

Answer: (b)

Solution:

$$I=\int_0^{\pi}\frac{dx}{1-2a\cos x+a^2} \qquad I=\int_0^{\pi}\frac{dx}{1+2a\cos x+a^2}$$

$$2I=\int_0^{\pi}\frac{2(1+a^2)}{(1+a^2)^2-4a^2\cos^2 x}$$

$$2I=2\int_0^{\frac{\pi}{2}}\frac{2(1+a^2)}{(1+a^2)^2-4a^2\cos^2 x}$$

$$I=\int_0^{\frac{\pi}{2}}\frac{2(1+a^2)\sec^2 x}{(1+a^2)^2(1+\tan^2 x)-4a^2}dx$$

$$I=\int_0^{\frac{\pi}{2}}\frac{\left(\frac{2}{1+a^2}\right)\sec^2 x}{\tan^2 x+\left(\frac{1-a^2}{1+a^2}\right)}dx$$

Put $\tan x = t$

$$I = \frac{2}{1+a^2}\int_0^{\infty}\frac{dt}{t^2+\left(\frac{1-a^2}{1+a^2}\right)^2}$$

$$=\left(\frac{2}{1+a^2}\right)\times\left(\frac{1+a^2}{1-a^2}\right)\times\left|\tan^{-1} t\frac{(1+a^2)}{(1-a^2)}\right|_0^{\infty}$$

$$=\frac{2}{1-a^2}\times\frac{\pi}{2}$$

$$=\frac{\pi}{1-a^2}$$

Question: If the area $\{(x, y): 0 \le y \le \min(2x, 6x-x^2)\}$ is A then 12 A ____.

Answer: 304.00

Solution:

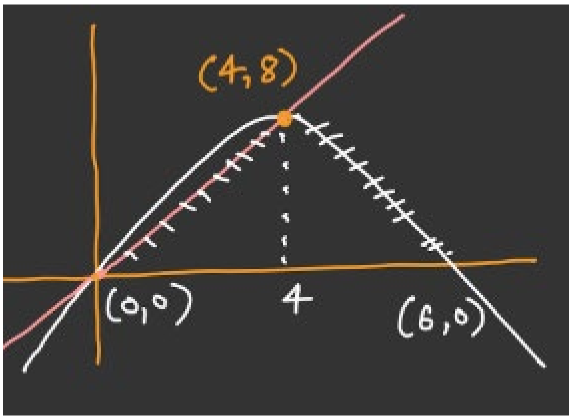
Given, $0 \le y \le \min(2x, 6x-x^2)$

$6x - x^2 = 2x$

$x^2 = 4x$

$x = 0,\ x = 4$

$$A = 16 + \int_4^6 (6x - x^2)\,dx$$



$$A = 16 + \left[3x^2 - \frac{x^3}{3}\right]_4^6$$

$12A = 304$

Question: If mean of 15 observation is 12 & s.d. = 3. 10 was taken wrongly in place of 12. New mean = μ, Variance = σ^2. Find the value $=15\left(\mu+\mu^2+\sigma^2\right)$

Answer: 2521.00

Solution:

$$\frac{\left(x_1+x_2+...+x_{14}\right)+10}{15}=12 \quad ....(1)$$

$$\frac{\left(x_1+x_2+....+x_{14}\right)+12}{15}=\mu \quad ...(2)$$

(2) – (1)

$$\frac{2}{15}=\mu-12$$

$$\mu=\frac{182}{15}$$

$$\frac{x_1{}^2+...+x_{14}{}^2+10^2}{15}-12^2=9 \quad ...(3)$$

$$\frac{x_1{}^2+....+x_{14}{}^2+12^2}{15}-\left(\frac{182}{15}\right)^2=\sigma^2 \quad ...(4)$$

(4) – (3)

$$\left(\frac{12^2-10^2}{15}\right)-\left(\left(\frac{182}{15}\right)^2-12^2\right)=\sigma^2-9$$

$$\frac{44}{15}-\left(\frac{182}{15}\right)^2+144=\sigma^2-9$$

$$\left(\frac{182}{15}\right)^2+\sigma^2=\frac{44}{15}+144+19$$

$$\mu^2+\sigma^2+\mu=\frac{44}{15}+153+\frac{182}{15}$$

$$15\left(\mu+\mu^2+\sigma^2\right)=44+182+153\times15=2521$$

Question: $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix}=0$ then α lies in

Options:

(a) $\left[-\frac{3}{2},\frac{3}{2}\right]$

(b) $\left(-3,0\right)$

(c)

(d)

Answer:

Solution:

$$\left(\alpha+\frac{3}{2}\right)\left[3\alpha+1-\frac{2\alpha}{3}-1\right]-\left(\alpha+\frac{1}{3}\right)\left[3\alpha+1-3\alpha-\frac{9}{2}\right]=0$$

$$\left(\alpha+\frac{3}{2}\right)\frac{7\alpha}{3}+\left(\alpha+\frac{1}{3}\right)\frac{7}{2}=0$$

$$\frac{\alpha^2}{3}+\frac{\alpha}{2}+\frac{\alpha}{2}+\frac{1}{6}=0$$

$$2\alpha^2+6\alpha+1=0$$

Question: Find the number of solutions: $\tan^{-1}(x)+\tan^{-1}(2x)=\frac{\pi}{4}$

Answer: 1.00

Question: A-m elements, B-n elements, subset of A is 56 more than B, P(m, n) is a point and Q(−2, −3), find distance between P and Q.
Answer: –10.00

Question: $\alpha=\frac{(4!)!}{(4!)^{3!}}$ & $\beta=\frac{(5!)!}{(5!)^{4!}}$

Options:
(a) α is integer β is not
(b) β is integer α is not
(c) Both are integers
(d) Both are not integers

Question: Let A be 2 × 2 real matrix and roots of equation $|A-x|=0$ be −1 and 3. Sum of diagonal elements of A^2 ____.

Question: $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)$ and $f''(x)>0$ for $x\in(0,3)$