JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 22nd JANUARY 2025) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. The number of non-empty equivalence relations on the set {1,2,3} is :

(1) 6 (2) 7

(3) 5 (4) 4

Ans. (3)

Sol. Let R be the required relation

A = {(1, 1) (2, 2), (3, 3)}

(i) | R | = 3, when R = A

(ii) | R | = 5, e.g. R = A ∪ {(1, 2), (2, 1)}

Number of R can be [3]

(iii) R = {1, 2, 3} × {1, 2, 3}

Ans. (5)

2. Let f : **R**→**R** be a twice differentiable function such that $f(x + y) = f(x)\,f(y)$ for all x, y ∈ **R**. If $f'(0) = 4a$ and f staisfies $f''(x) - 3a\,f'(x) - f(x) = 0$, a > 0, then the area of the region

$R = \{(x,y) \mid 0 \le y \le f(ax),\ 0 \le x \le 2\}$ is :

(1) $e^2 - 1$ (2) $e^4 + 1$

(3) $e^4 - 1$ (4) $e^2 + 1$

Ans. (1)

Sol. $f(x + y) = f(x).f(y)$

$\Rightarrow f(x) = e^{\lambda x} \quad f'(0) = 4a$

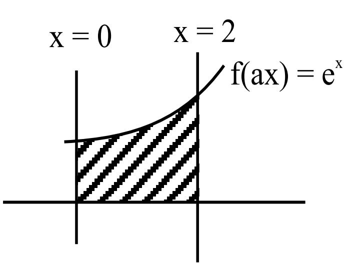
$\Rightarrow f'(x) = \lambda e^{\lambda x} \Rightarrow \lambda = 4a$

So, $f(x) = e^{4ax}$

$f''(x) - 3af'(x) - f(x) = 0$

$\Rightarrow \lambda^2 - 3a\lambda - 1 = 0$

$\Rightarrow 16a^2 - 12a^2 - 1 = 0 \Rightarrow 4a^2 = 1 \Rightarrow \boxed{a = \frac{1}{2}}$



$F(x) = e^{2x}$

$\text{Area} = \int_0^2 e^x dx = \boxed{e^2 - 1}$

3. Let the triangle PQR be the image of the triangle with vertices (1,3), (3,1) and (2, 4) in the line x + 2y = 2. If the centroid of ΔPQR is the point (α, β), then 15(α – β) is equal to :

(1) 24 (2) 19

(3) 21 (4) 22

Ans. (4)

Sol. Let 'G' be the centroid of Δ formed by (1, 3) (3, 1) & (2, 4)

$G \cong \left(2, \frac{8}{3}\right)$

Image of G w.r.t. x + 2y – 2 = 0

$\frac{\alpha - 2}{1} = \frac{\beta - \frac{8}{3}}{2} = -2\frac{\left(2 + \frac{16}{3} - 2\right)}{1+4}$

$= \frac{-2}{5}\left(\frac{16}{3}\right)$

$\Rightarrow \alpha = \frac{-32}{15} + 2 = \frac{-2}{15},\ \beta = \frac{-32 \times 2}{15} + \frac{8}{3} = \frac{-24}{15}$

$15(\alpha - \beta) = -2 + 24 = 22$

4. Let z_1, z_2 and z_3 be three complex numbers on the circle $|z| = 1$ with $\arg(z_1) = \frac{-\pi}{4}$, $\arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$.

If $\left|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right|^2 = \alpha + \beta\sqrt{2}$, α, β ∈ Z, then the value of $\alpha^2 + \beta^2$ is :

(1) 24 (2) 41

(3) 31 (4) 29

Ans. (4)

Sol. $Z_1 = e^{-i\pi/4}$, $Z_2 = 1$, $Z_3 = e^{i\pi/4}$

$\left|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right|^2 = \left|e^{-i\frac{\pi}{4}} \times 1 + 1 \times e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}} \times e^{i\frac{\pi}{4}}\right|^2$

$\left|e^{-i\frac{\pi}{4}} + e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}}\right|^2$

$= \left|2e^{-i\frac{\pi}{4}} + i\right|^2 = \left|\sqrt{2} - \sqrt{2}i + i\right|^2$

$= \left(\sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 2 + 1 + 2 - 2\sqrt{2} = 5 - 2\sqrt{2}$

$\alpha = 5, \beta = -2$

$\Rightarrow \alpha^2 + \beta^2 = 29$

5. Using the principal values of the inverse trigonometric functions the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2 + (\operatorname{cosec}^{-1}x)^2)$ is :

(1) $24\pi^2$ (2) $18\pi^2$

(3) $31\pi^2$ (4) $22\pi^2$

Ans. (4)

Sol. $16(\sec^{-1}x)^2 + (\operatorname{cosec}^{-1}x)^2$

$$\operatorname{Sec}^{-1}x = a \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - a$$

$$= 16\left[a^2 + \left(\frac{\pi}{2} - a\right)^2\right] = 16\left[2a^2 - \pi a + \frac{\pi^2}{4}\right]$$

$$\max]_{a=\pi} = 16[2\pi^2 - \pi^2 + \pi\frac{2}{4}] = 20\pi^2$$

$$\min]_{a=\frac{\pi}{4}} = 16\left[\frac{2\times\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4}\right] = 2\pi^2$$

Sum $= 22\pi^2$

6. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64(\mu + \sigma^2)$ is :

(1) 51 (2) 48

(3) 32 (4) 64

Ans. (2)

Sol. HHH $\to$ 0

HHT $\to$ 0

HTH $\to$ 1

HTT $\to$ 0

THH $\to$ 1

THT $\to$ 1

TTH $\to$ 1

TTT $\to$ 0

Probability distribution

x_i	0	1
$P(x_i)$	$1/2$	$1/2$

$$\mu = \sum x_i p_i = \frac{1}{2}$$

$$\sigma^2 = \sum x_i^2 p_i - \mu^2$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 48$$

7. Let a_1, a_2, a_3..... be a G.P. of increasing positive terms. If $a_1a_5 = 28$ and $a_2 + a_4 = 29$, the a_6 is equal to

(1) 628 (2) 526

(3) 784 (4) 812

Ans. (3)

Sol. $a_1.a_5 = 28 \Rightarrow a.ar^4 = 28 \Rightarrow a^2r^4 = 28$ …(1)

$a_2 + a_4 = 29 \Rightarrow ar + ar^3 = 29$

$\Rightarrow ar(1 + r^2) = 29$

$\Rightarrow a^2r^2(1 + r^2)^2 = (29)^2$ …(2)

By Eq. (1) & (2)

$$\frac{r^2}{(1+r^2)^2} = \frac{28}{29\times 29}$$

$$\Rightarrow \frac{r}{1+r^2} = \frac{\sqrt{28}}{29} \Rightarrow r = \sqrt{28}$$

$$\because a^2r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28$$

$$\Rightarrow a = \frac{1}{\sqrt{28}}$$

$$\therefore a_6 = ar^5 = \frac{1}{\sqrt{28}} \times (28)^2\sqrt{28} = 784$$

8. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and

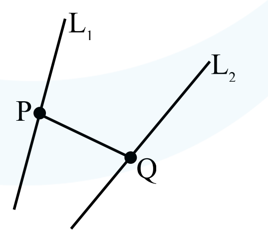
$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

(1) $\left(-\frac{5}{3}, -7, 1\right)$ (2) $\left(2, 3, \frac{1}{3}\right)$

(3) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$ (4) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Ans. (4)

Sol.



$P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ on L_1

$Q(3\mu + 2, 4\mu + 4, 5\mu + 5)$ on L_2

Dr's of PQ $= 3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2$

$PQ \perp L_1$

$\Rightarrow (3\mu - 2\lambda + 1)2 + (4\mu - 3\lambda + 2)3 + (5\mu - 4\lambda + 2)4 = 0$

$38\mu - 29\lambda + 16 = 0$ ...(1)

$PQ \perp L_2$

$\Rightarrow (3\mu - 2\lambda + 1)3 + (4\mu - 3\lambda + 2)4 + (5\mu - 4\lambda + 2)5 = 0$

$50\mu - 38\lambda + 21 = 0$ ...(2)

By (1) & (2)

$\lambda = \frac{1}{3};\ \mu = \frac{-1}{6}$

$\therefore\ P\left(\frac{5}{3}, 3, \frac{13}{3}\right)\ \&\ Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Line PQ

$$\frac{x - \frac{5}{3}}{\frac{1}{6}} \quad \frac{y-3}{\frac{-1}{3}} \quad \frac{z - \frac{13}{3}}{\frac{1}{6}}$$

$$\frac{x - \frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z - \frac{13}{3}}{1}$$

Point $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

lies on the line PQ

9. The product of all solutions of the equation $e^{5(\log_e x)^2 + 3} = x^8$, $x > 0$, is :

(1) $e^{8/5}$ (2) $e^{6/5}$

(3) e^2 (4) e

Ans. (1)

Sol. $e^{5(\ell nx)^2 + 3} = x^8$

$\Rightarrow \ell n\, e^{5(\ell nx)^2 + 3} = \ell n\, x^8$

$\Rightarrow 5(\ell nx)^2 + 3 = 8\ell nx$

$(\ell nx = t)$

$\Rightarrow 5t^2 - 8t + 3 = 0$

$t_1 + t_2 = \frac{8}{5}$

$\ell n x_1 x_2 = \frac{8}{5}$

$x_1 x_2 = e^{8/5}$

10. If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{T_r}\right)$ is equal to :

(1) 1 (2) 0

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Ans. (3)

Sol. $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = \frac{1}{8}(2n-1)(2n+1)(2n+3)$

$\Rightarrow \frac{1}{T_n} = \frac{8}{(2n-1)(2n+1)(2n+3)}$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{T_r} = \lim_{n\to\infty} 8 \sum_{r=1}^{n} \frac{1}{(2n-1)(2n+1)(2n+3)}$$

$$= \lim_{n\to\infty} \frac{8}{4} \sum \left(\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right)$$

$$= \lim_{n\to\infty} 2\left[\left(\frac{1}{1.3} - \frac{1}{3.5}\right) + \left(\frac{1}{3.5} - \frac{1}{5.7}\right) + \ldots\right]$$

$$= \frac{2}{3}$$

11. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is :

(1) 14950 (2) 6084

(3) 4356 (4) 5148

Ans. (4)

Sol. $\underbrace{AB\ \_\_\_\_}_{12}\ M\ \underbrace{N.........Z}_{13}$

$= \underbrace{{}^{12}C_2}_{\text{Selection of two letters before M}} \times \underbrace{{}^{13}C_2}_{\text{Selection of two letters after M}} = 5148$

12. Let $x = x(y)$ be the solution of the differential equation $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If $x(1) = 1$, then $x\left(\frac{1}{2}\right)$ is :

(1) $\frac{1}{2} + e$ (2) $\frac{3}{2} + e$

(3) $3 - e$ (4) $3 + e$

Ans. (3)

Sol. $\frac{dx}{dy}+\left(\frac{1}{y^2}\right)x=\frac{1}{y^3}$

$\text{I.F.}=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$

$\Rightarrow x.e^{-\frac{1}{y}}=\int\left(e^{-\frac{1}{t}}\right).\frac{1}{y^3}dy$

Put $-\frac{1}{y}=t$

$+\frac{1}{y^2}dy=dt$

$x.e^{-\frac{1}{y}}=-\int t.e^t dt$

$x.e^{-\frac{1}{y}}=-te^t+e^t+C$

$x.e^{-\frac{1}{y}}=\frac{+1}{y}e^{-\frac{1}{y}}+e^{-\frac{1}{y}}+C$

$x=1,\ y=1$

$\frac{1}{e}=\frac{1}{e}+\frac{1}{e}+C$

$\Rightarrow C=-\frac{1}{e}$

Put $y=\frac{1}{2}$

$\frac{x}{e^2}=\frac{2}{e^2}+\frac{1}{e^2}-\frac{1}{e}$

$x=3-e$

13. Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R. If the circle C with centre at (–1, –1) passes through the points P, Q and R, then the area of ΔPQR is :

(1) 4 (2) 6

(3) 7 (4) 5

Ans. (2)

Sol. $y=x^2+px-3$

Let $P(\alpha, 0)$, $Q(\beta, 0)$, $R(0, -3)$

Circle with centre (–1, –1) is $(x+1)^2+(y+1)^2=r^2$

Passes through (0, –3)

$1^2+(-2)^2=r^2$]

$r^2=5$

$(x+1)^2+(y+1)^2=5$

Put y = 0

$(x+1)^2=5-1$

$(x+1)^2=4$

$x+1=\pm 2$

$x=1$ or $x=-3$

$\therefore$ P(1, 0) and Q(–3,0)

Area of $\Delta PQR=\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ -3 & 0 & 1\\ 0 & -3 & 1\end{vmatrix}=6$

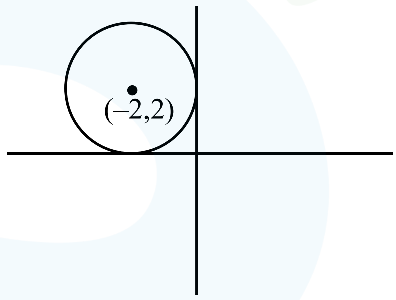
14. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point (2, 5) and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta-2\alpha$ is equal to :

(1) 15 (2) 14

(3) 12 (4) 10

Ans. (1)

Sol.



$S_1 : (x+2)^2+(y-2)^2=2^2$

$S_2 : (x-2)^2+(y-5)^2=r^2$

Both circle intersect at two points

$\therefore |r_1-r_2|<c_1c_2<r_1+r_2$

$|r-2|<5<2+r$

$\Rightarrow 3<r<7$

$r\in(3, 7)$

$\alpha=3, \beta=7$

$3\beta-2\alpha=15$

15. Let for $f(x) = 7\tan^8x + 7\tan^6x - 3\tan^4x - 3\tan^2x$, $I_1=\int_0^{\pi/4} f(x)dx$ and $I_2=\int_0^{\pi/4} x f(x)dx$. Then $7I_1+12I_2$ is equal to :

(1) 2π (2) π

(3) 1 (4) 2

Ans. (3)

Sol. $f(x) = (7\tan^6 x - 3\tan^2 x)(\sec^2 x)$

$$I_1 = \int_0^{\pi/4} (7\tan^6 x - 3\tan^2 x)(\sec^2 x)\,dx$$

Put tanx = t

$$I_1 = \int_0^1 (7t^6 - 3t^2)\,dt = \left[t^7 - t^3\right]_0^1 = 0$$

$$I_2 = \int_0^{\pi/4} \underset{I}{x}\underbrace{(7\tan^6 x - 3\tan^2 x)(\sec^2 x)}_{II}\,dx$$

$$= \left[x\left(\tan^7 x - \tan^3 x\right)\right]_0^{\pi/4} - \int_0^{\pi/4} (\tan^7 x - \tan^3 x)\,dx$$

$$= 0 - \int_0^{\pi/4} \tan^3 x\left(\tan^2 x - 1\right)\left(1 + \tan^2 x\right)dx$$

Put tanx = t

$$= -\int_0^1 \left(t^5 - t^3\right)dt = -\left[\frac{t^6}{6} - \frac{t^4}{4}\right] = \frac{1}{12}$$

$7I_1 + 12I_2 = 1$

16. Let $f(x)$ be a real differentiable function such that $f(0) = 1$ and $f(x + y) = f(x)f'(y) + f'(x)\,f(y)$ for all $x, y \in \mathbf{R}$. Then $\sum_{n=1}^{100} \log_e f(n)$ is equal to :

(1) 2384 (2) 2525

(3) 5220 (4) 2406

Ans. **(2)**

Sol. $f(x + y) = f(x)\,f'(y) + f'(x)\,f(x)$

Put = x = y = 0

$f(0) = f(0)f'(0) + f'(0)f(0)$

$f'(0) = \dfrac{1}{2}$

Put y = 0

$f(x) = f(x)\,f'(0) + f'(x)f(0)$

$$f(x) = \frac{1}{2}f(x) + f'(x)$$

$$f'(x) = \frac{f(x)}{2}$$

$$\frac{dy}{dx} = \frac{y}{2} \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{2}$$

$$\Rightarrow \ell ny = \frac{x}{2} + c$$

$\because f(0) = 1 \Rightarrow C = 0$

$$\ell ny = \frac{\pi}{2} \Rightarrow f(x) = e^{x/2}$$

$$\ell n\, f(n) = \frac{n}{2}$$

$$\sum_{n=1}^{100} \ell f(n) = \frac{1}{2}\sum_{n=1}^{100} n = \frac{5050}{2}$$

= 2525

17. Let A = {1,2,3,.......,10} and

$$B = \left\{\frac{m}{n} : m, n \in A,\ m < n \text{ and } \gcd(m, n) = 1\right\}.$$

Then n(B) is equal to :

(1) 31 (2) 36

(3) 37 (4) 29

Ans. **(1)**

Sol. A = {1, 2, ….10}

$B\ \{\frac{m}{n} = m, n \in A, m < n, \gcd(m, n) = 1\}$

n(B)

n = 2 $\left\{\frac{1}{2}\right\}$

n = 3 $\left\{\frac{1}{3}, \frac{2}{3}\right\}$

n = 4 $\left\{\frac{1}{4}, \frac{3}{4}\right\}$

n = 5 $\left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$

n = 6 $\left\{\frac{1}{6}, \frac{5}{6}\right\}$

n = 7 $\left\{\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right\}$

n = 8 $\left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}$

n = 9 $\left\{\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}\right\}$

n = 10 $\left\{\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}\right\}$

n(B) = 31

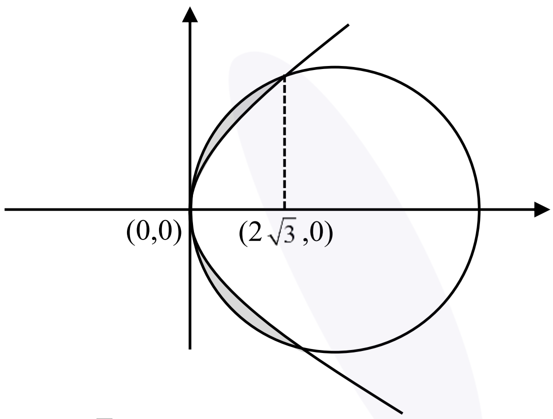
18. The area of the region, inside the circle $\left(x-2\sqrt{3}\right)^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is

(1) $6\pi-8$ (2) $3\pi-8$

(3) $6\pi-16$ (4) $3\pi+8$

Ans. (3)

Sol.



$y^2 = 2\sqrt{3}x$

$\left(x-2\sqrt{3}\right)^2+y^2=\left(2\sqrt{3}\right)^2$

$A=\frac{\pi r^2}{2}-2\int_0^{2\sqrt{3}}\sqrt{2\sqrt{3}x}\,dx$

$\frac{\pi(12)}{2}-2\sqrt{2\sqrt{3}}\frac{\left(x^{3/2}\right)_0^{2/3}}{3/2}$

$= 6\pi-16$

19. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to :

(1) 14 (2) 4

(3) 11 (4) 13

Ans. (1)

Sol. $P=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{4}{10}\times\frac{6}{9}+\frac{6}{10}\times\frac{5}{9}}=\frac{5}{9}$

$m = 5, n = 9$

$m + n = 14$

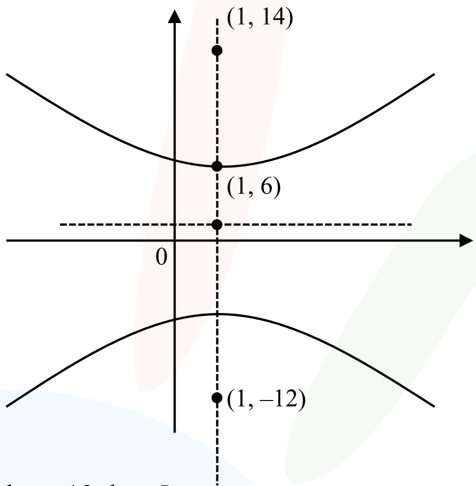
20. Let the foci of a hyperbola be (1, 14) and (1, –12). If it passes through the point (1, 6), then the length of its latus-rectum is :

(1) $\frac{25}{6}$ (2) $\frac{24}{5}$

(3) $\frac{288}{5}$ (4) $\frac{144}{5}$

Ans. (3)

Sol.



$be = 13, b = 5$

$a^2 = b^2(e^2-1)$

$= b^2e^2-b^2$

$= 169 - 25 = 144$

$\ell(LR)=\frac{2a^2}{b}=\frac{2\times144}{5}=\frac{288}{5}$

SECTION-B

21. Let the function,

$f(x)=\begin{cases}-3ax^2-2, & x<1\\ a^2+bx, & x\ge1\end{cases}$

Be differentiable for all $x\in\mathbf{R}$, where $a > 1$, $b\in\mathbf{R}$. If the area of the region enclosed by $y = f(x)$ and the line $y = -20$ is $\alpha+\beta\sqrt{3}$, $\alpha, \beta, \in Z$, then the value of $\alpha+\beta$ is ____.

Ans. (34)

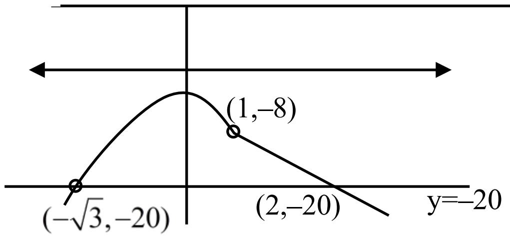
Sol. f(x) is continuous and differentiable

at x = 1; LHL = RHL, LHD = RHD

$-3a-2=a^2+b, -6a=b$

$a = 2, 1;\ b = -12$

$f(x)=\begin{cases}-6x^2-2 & ; x<1\\ 4-12x & ; x\ge1\end{cases}$



$$\text{Area} = \int_{-\sqrt{3}}^{1}\left(-6x^2-2+20\right)dx+\int_{1}^{2}(4-12x+20)\,dx$$

$$16+12\sqrt{3}+6=22+12\sqrt{3}$$

22. If $\sum_{r=0}^{5}\frac{{}^{11}C_{2r+1}}{2r+2}=\frac{m}{n}$, gcd(m, n) =1, then m – n is equal to ______.

Ans. (2035)

Sol. $\int_{0}^{1}(1+x)^{11}dx=\left[C_0x+\frac{C_1x^2}{2}+\frac{C_2x^3}{3}+\ldots\right]_0^1$

$$\frac{2^{12}-1}{12}=C_0+\frac{C_1}{2}+\frac{C_2}{3}+\frac{C_3}{4}+\ldots$$

$$\int_{-1}^{0}(1+x)^{11}dx=\left[C_0x+\frac{C_1x^2}{2}+\frac{C_2x^3}{3}+\ldots\right]_{-1}^{0}$$

$$\frac{1}{12}=C_0-\frac{C_1}{2}+\frac{C_2}{3}-\frac{C_3}{4}+\ldots$$

$$\frac{2^{12}-2}{12}=2\left(\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+\ldots\right)$$

$$\frac{C_1}{2}+\frac{C_3}{4}-\frac{C_5}{6}+\ldots=\frac{2^{11}-1}{12}=\frac{2047}{12}$$

23. Let A be a square matrix of order 3 such that det(A) = –2 and det(3adj(–6adj(3A))) $=2^{m+n}.3^{mn}$, m > n. Then 4m + 2n is equal to _____.

Ans. (34)

Sol. $|A| = -2$

det(3adj(–6adj(3A)))

$= 3^3\det(\text{adj}(-\text{adj}(3A)))$

$= 3^3(-6)^6(\det(3A))^4$

$= 3^{21}\times 2^{10}$

$m + n = 10$

$mn = 21$

$m = 7\ ;\ n = 3$

24. Let $L_1:\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and

$L_2:\frac{x-2}{2}=\frac{y}{0}=\frac{z+4}{\alpha}$, $\alpha\in R$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point A(1, 1, –1) on L_2, then the value of $26\,\alpha(PB)^2$ is ____.

Ans. (216)

Sol. Point B

$(3\lambda+1, -\lambda+1, -1)\equiv(2\mu+2, 0, \alpha\mu-4)$

$3\lambda+1=2\mu+2$

$-\lambda+1=0$

$-1=\alpha\mu-4$

$\lambda=1, \mu=1, \alpha=3$

B(4, 0, –1)

Let Point 'P' is $(2\delta+2, 0, 3\delta-4)$

Dr's of AP $< 2\delta+1, -1, 3\delta-3 >$

$AP\perp L_2\Rightarrow \delta=\frac{7}{13}$

$$P\left(\frac{40}{13},0,\frac{-31}{13}\right)$$

$$2\sigma\delta(PB)^2=26\times3\times\left(\frac{144}{169}+\frac{324}{169}\right)$$

$= 216$

25. Let $\vec{c}$ be the projection vector of $\vec{b}=\lambda\hat{i}+4\hat{k}$, $\lambda>0$, on the vector $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$. If $|\vec{a}+\vec{c}|=7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is ______.

Ans. (16)

Sol. $\vec{c}=\left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$

$$=\left(\frac{\lambda+8}{9}\right)\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

$|\vec{a}+\vec{c}|-=7\Rightarrow\lambda=4$

Area of parallelogram

$$=\left|\vec{b}\times\vec{c}\right|=\left|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\ \frac{4}{3}&\frac{8}{3}&\frac{8}{3}\\ 4&0&4\end{vmatrix}\right|$$

$= 16$